

Week 9 Notes

For Recap:

Because it is easy to lose the forest in the trees, I want to summarize the big ideas from Ulf's presentation last week.

I. Conceptual Progress

Background:

In my summary of four nested, increasingly expressively powerful approaches to semantics last time, I compared them along these lines:

- i) universe, from which is drawn or constructed
- ii) semantic interpretants assigned to sentences,
- iii) in terms of which reason relations of consequence and incompatibility-incoherence are defined.
- iv) The *interpretation function* takes one from (i) to (ii), assigning each sentence some structure constructed or computed from elements of the universe as its semantic interpretant.
- v) The *semantic definition of consequence* (and incompatibility) then takes one from (ii) to (iii).

Possible worlds semantics and Tarskian model-theoretic semantics are alike in that they (iv) take semantic interpretants (ii) to be *sets* of elements of their universes (possible worlds or models = relational structures, respectively) and (v) define consequence and incompatibility (iii) by set-theoretic relations of *inclusion* and *exclusion* (disjointness).

(The additional Kripkean accessibility-structure imposed on the universe of possible worlds is used in assigning semantic interpretants to *modal* sentences—those formed using the necessity operator \Box or the possibility operator \Diamond .)

Ulf's First Big Idea (Critical):

Ulf noticed that in defining reason relations, in particular, implication or consequence, in the truthmaker semantic framework (v), Kit Fine only uses *one* of the three additional structural elements he has added to the more traditional possible worlds / model-theoretic framework.

Those three new elements are:

1. Using a universe of *states* with a *mereological*, part-whole structure imposed on it, from which to draw semantic interpretants that will be assigned to sentences in order to codify

the reason relations those sentences stand in (which I have claimed is the defining task of semantics).

2. Partitioning the universe of states into two disjoint regions: *possible* states and *impossible* states. (This is further structure imposed on the universe of states, in addition to the mereological order imposed by the “fusion” operation that forms wholes from parts.)
3. Using as *semantic interpretants* (assigned to sentences by the interpretation function) ordered *pairs* of sets of states (drawn from the mereologically structured universe of states), understood as the truthmakers and falsemakers (verifiers/falsifiers) of those sentences.

Fine mostly works with two ways of defining consequence or implication in this semantic setting:

What he calls “entailment” just applies the strategy common to possible worlds and model-theoretic semantics in this new, richer setting. It appeals to set-theoretic inclusion relations among the sets of truthmakers only. $\Gamma \sim A$ in this sense just in case all the truthmakers of (everything in) Γ are also truthmakers of A .

What he calls “containment” (not to be confused with the structural principle in Gentzen-style proof-theoretic settings that generalizes Reflexivity) appeals in addition to the mereological structure on the universe from which semantic interpretants are drawn.

(On this definition, $B \sim A$ just in case every truthmaker of A contains as a part a truthmaker of B , and every truthmaker of B is part of a truthmaker of A .)

This notion of consequence or implication is unique to the truthmaker framework, depending as it does on the mereological structure of the universe of state, and it does a lot of good expressive work for Fine.

But, Ulf noticed, these semantic definitions of reason relations appeal *only* to truthmakers.

They do not at all use the *bipolar* or *bivalent* character of semantic interpretants as consisting not only of *truthmakers* but also of *falsemakers*—feature (3) above.

The definition of consequence as containment *does* use the mereological structure on the universe of states—feature (1) above—though consequence understood as entailment does not. But neither definition appeals to the further *modal* structure of the universe of states that consists in its being divided exclusively and exhaustively into possible and impossible states.

These additional semantic resources provided by the truthmaker setting are *not* being exploited to provide any kind of additional expressive power as far as definitions of reason relations are concerned.

This critical observation about the minimal, cautious use Fine makes of the riches his truthmaker framework affords for defining the reason relations we take to be central to the conceptual content expressed by sentences invites a constructive response: see what reason relations of consequence and incompatibility are definable if we *do* avail ourselves of the full conceptual resources Fine has made available.

Ulf's Second Big Idea (Constructive):

Here Ulf had a fabulous insight—the sort of conceptual *coup de foudre* philosophers live for: The two elements of the truthmaker framework that Fine was *not* exploiting correspond exactly to the two basic components of Restall and Ripley's bilateralist normative pragmatic understanding of reason relations.

- Their distinction between assertions and denials (speech acts corresponding to practical attitudes of acceptance and rejection) lines up with Fine's distinction between truthmakers and falsemakers (verifiers and falsifiers)—point (3) above.
- And their normative distinction between deontic positions that are *out-of-bounds* and *in-bounds* (what gets rendered in the Brandom-Simonelli version of RR-bilateralism as constellations of commitments to which one *cannot* be jointly entitled and those constellations of commitments to which one *can* be jointly entitled) lines up with Fine's distinction between *impossible* and *possible* states—point (2) above.

Ulf realized that that analogy could be sharpened into a translation or transliteration of the bilateralist definition of the implication relations expressed by multisuccedent sequents in Gentzen-style proof-theoretic frameworks into a definition of implication relations in Fine's truth-maker semantic framework.

Restall and Ripley's definition of consequence is:

$\Gamma \vdash \Delta$ iff *asserting every element of Γ and denying every element of Δ is out of bounds.*

The result is Ulf's proposed definition of consequence:

$\Gamma \vdash \Delta$ iff **every fusion of truthmakers of every sentence in Γ with falsemakers of every sentence in Δ is an impossible state.**

This way of understanding consequence makes use of all three fundamental elements that distinguish Fine's truthmaker setting from possible worlds semantics:

From (1): The *mereological* structure of the universe of states is appealed to in looking at *fusions* of truthmakers of premises and falsemakers of conclusions.

From (2): The *modal* structure of the universe of states is used to implement an analogue of Restall and Ripley's "out-of-boundness."

From (3): The bipolar character of the semantic interpretants of sentences in the truthmaker framework is exploited through the analogy with Restall-Ripley bilateralism, in that both truthmakers (of premises) and falsemakers (of conclusions) are essential to the definition.

II. Technical Progress

Ulf's First Result:

For various purposes, Fine sometimes imposes conditions regarding the interaction of the mereological structure of the state space with its modal structure.

Since his semantic definitions of different consequence relations (“entailment,” “containment”) do not depend on the modal division of the state space into possible and impossible states, these conditions do not affect the reason relations he can define.

Because Ulf's translation of the normative bilateralist understanding of consequence into the truthmaker framework *does* appeal to the modal partition of the mereological state space into possible/impossible states, those conditions *do* affect the consequence (and incompatibility) relations Ulf defines.

Fine's conditions are Downward Closure, Exhaustivity, and Exclusivity.

Considering the effects of imposing these conditions on consequence as Ulf defines it, it is clear immediately that **Downward Closure** (all the parts of possible states are possible states, or equivalently, all wholes containing impossible parts are impossible) corresponds to the **Monotonicity** (MO) of implication. If $\Gamma \sim \Delta$, then any fusion of the truthmakers of Γ with the falsmakers of Δ must be impossible. Since if that is so, fusing those states with any other states at all must also be impossible. So it follows that $\Gamma, \Pi \sim \Theta, \Delta$

Fine's **Exclusivity** condition in effect imposes a Dummettian *harmony* condition on the interpretation of sentences, in the form of a metainferential structural principle concerning implication relations.

For any sentence A such that the interpretation function assigns the set of states X as A 's truthmakers and the set Y as A 's falsmakers, it must be the case that the fusion of all of X with all of Y must be impossible.

Given Ulf's bilateral definition of consequence, this is a representation of a kind of *implication* curled up in the two components of Fine's semantic interpretants.

Specifically, it is the requirement that $A \sim A$.

In the sequent calculus, this is the structural principle of **Reflexivity** (RE) of implication.

(In the context of Downward Closure, this condition means that for *any* sets of sentences Γ, Δ , it must the case that $\Gamma, A \sim A, \Delta$.

In the sequent calculus, this is the structural conditional (metainferential rule) we call “**Containment**” (CO)—not to be confused with Fine's use of that term for a kind of consequence relation.

CO is a special kind of monotonicity, and is implied by MO together with RE.)

The subtlest and most sophisticated insight Ulf then has concerning the relation between Fine's conditions on the interaction of mereological and modal structure in the truthmaker setting is that

his bilateral definition of consequence means that Fine’s **Exhaustivity** condition—once it is put in the right form, equivalent to the form Fine uses—imposes *transitivity* on the consequence relation, in the form of (shared context) Cut (or Cumulative Transitivity) (CT).

This fact—this part of the way Ulf lines up Fine’s conditions on the interaction of mereology and modality with Gentzen’s structural conditions—is, I think, not at all obvious.

It depends on using the isomorphism Ulf develops to adapt Ripley’s account of the significance of transitivity (Cut) on a bilateralist understanding of sequents to Fine’s truthmaker semantic framework.

Here is how I understand Ulf’s argument, broken down into pieces.

Exhaustivity: $\forall u \in S\Diamond$, either $\exists s \in |p|_+ (u \sqcup s \in S\Diamond)$ or $\exists t \in |p|_- (u \sqcup t \in S\Diamond)$.

Ulf added the reference to u .

It is redundant: By Downward Closure, if the fusion of u with s (or t) is possible, so are all its substates.

The two formulations are accordingly equivalent.

But, Ulf argues, this way of putting it is more revealing.

For it makes clear the sense in which it is the semantic expression of Cut (CT).

For that *reversible* rule is this:

$$\frac{\Gamma \sim A, \Delta \quad \Gamma, A \sim \Delta}{\Gamma \sim \Delta}$$

We read $\Gamma \sim \Delta$ as it’s being out of bounds to assert all of Γ and deny all of Δ .

That corresponds in truthmaker terms to: it is impossible that all of Γ be true and all of Δ be false. Γ is s , and Δ is t . That is what is below the line.

But the rule is reversible. So if that holds, then what is above the line must hold, as well.

That is, it must be the case that for *any* A —which is u —then one can either assert or deny A in the context of Γ and Δ . The impossibility = out-of-boundness is preserved, for *any* A (u).

One must formulate Exclusivity in this way to see the parallel.

It does not appear if you use the simpler equivalent form.

This is a *very* nice point.

Fine himself does not, I think, recognize this fact about Exhaustivity: the sense in which it is not just a kind of transitivity, but specifically, CT.

He cannot, because his definition of consequence does not depend on the modal partition of states into possible/impossible, which is what his structural conditions address.

Ripley certainly appreciates a specification like this of the significance of transitivity (which he wants to deny) in connection with the paradoxes.

Transitivity means in bilateralist terms that for *any* sentence, one can *either* assert it *or* deny it. He wants to say that that is not so for liar-type sentences—and *only* for them (nontransitivity is carefully limited to them). Those one can *neither* assert nor deny.

Transitivity, accordingly, precludes truth-value *gaps*.

Truth-value *gluts*—contradictions, in the sense of sentences that have the very same state as both truthmaker and falsemaker, and so can be both true and false—are prohibited by Exhaustivity, via RE, the requirement that $A \mid\sim A$. Ripley does not need to deny *that*.

So

| | Gentzen’s Sequent Structural Rules | Fine’s Modal Structural Conditions |
|--|--|---|
| | Monotonicity (MO) | Downward Closure |
| | Cumulative Transitivity = Cut (CT) | Exhaustiveness |
| | Reflexivity (RE). (RE+MO \Rightarrow CO) | Exclusivity |

These structural mappings *only* hold if consequence is understood on Ulf’s definition, not on any of Fine’s. That is, they depend on his translation of the bilateralist deontic normative pragmatic understanding of implication into the truthmaker semantic vocabulary.

They permit us to understand something deep and important about the truthmaker semantic framework that is *only* visible if that framework is viewed through the lens provided by Ulf’s consequence relation.

Ulf’s Second Result:

This insight—realizing the structural isomorphism between reason relations as he defines them in the truthmaker semantic metavocabulary and reason relations couched in (either the Restall-Ripley or the Brandom-Simonelli version of) the bilateral deontic normative pragmatic metavocabulary—makes it possible for Ulf to do something that no-one has yet done in the truthmaker semantic setting.

Since we know how to make things work relaxing all the structural restrictions in the *logical* metavocabulary of the sequent calculus, using Ulf’s consequence relation means we can transfer those techniques and that apparatus over to the truthmaker setting.

That allows Ulf to define truthmaker semantics for a whole class of *nonmonotonic* and *nontransitive* (that is, open rather than closed) logics (individually or together).

That is a way of showing the robustness of the isomorphism he identifies. For it works even when we start manipulating the pieces (structural conditions).

But the big confirmation of the power and the value of the isomorphism is that Ulf uses it to solve an unsolved problem in the truthmaker framework (construct adequate semantics for a whole range of substructural logics, in a principled way) by reducing it to an already solved

problem. (Well, already solved by us.) For that “solved” problem is characterizing such logics in the sequent calculus. Then, by applying

- first the bilateralist “translation” or definition of the pragmatic significance of implications (expressed by sequents) and then
- his modalized bilateral definition of consequence = implication (itself inspired by the RR-bilateralist reading of the turnstile) in truthmaker terms: fusion of truthmakers of all of Γ and falsemakers of all of Δ is always an *impossible* state.

He can carry over our success with sequent calculi with open structure to the truthmaker framework, producing truthmaker semantics for those same radically substructural, but extremely well-behaved, logics.

Compare: Descartes exploiting the constructive isomorphism he secured between algebra and geometry to turn difficult geometrical problems into easy algebraic ones, and then transferring the solutions back to solve geometry problems that had gone unsolved by those who did not have analogs of Descartes’ isomorphism.

Ulf takes it that one key feature of his definition of consequence (really, the key to the whole enterprise) is that it is *modalized*, appealing to the partition of the universe of states into possible and impossible states. Fine does not do that. He does not want to modalize the non-modal parts of his logic. But that is a mistake—or anyway, it is an unnecessary expressive restriction. Or, rather, it imposes a high expressive price. We do not need to pay it.

But there is deeper conceptual point here. There is something modal about consequence, at least material consequence. Cf. Ryle “an aspect of generality” about any implication (“If today is Wednesday, tomorrow will be Thursday.”) And we should acknowledge that even if the *logic* we are going to introduce is expressively impoverished enough that it cannot *express* this modal character. We can introduce one that *can*—as Dan shows with NM-MS.

So there is an issue of conceptual and explanatory priority here: modality is wanted in the semantics, even for nonmodal classical logic.

And the fruitfulness of Ulf’s modalized semantic definition of consequence—for instance, in leading to solving the hitherto unsolved problem of giving a *recipe* for producing truthmaker semantics for a wide variety of substructural logics (well, NM-MS with a variety of kinds of openness of structure = substructurality)—speaks in favor of this order of explanation, this direction of conceptual priority and order of intelligibility as an *explanatory strategy*.

III. Philosophical Interpretation (of Ulf's Result)

[Bimodal Hylomorphic Conceptual Realism.

Two versions of the *content* that is common, that read the hylomorphic metaphor differently:

- a) One *content* specified in two *metavocabularies*: normative pragmatic and alethic semantic. (Bob's Hegel.)
- b) One *form* for two *matters*: mind and world. (Ulf's Aristotle.)

Wittgenstein says: "When we say, and mean, that such-and-such is the case, we—and our meaning—do not stop anywhere short of the fact; but we mean: this—is—so." [*PI* §95.]

We agree that Dan's implication-space semantics is a way to specify that *content* or *form* that is independent of the pragmatic and semantic metavocabularies, and the matter of mind or world.

- c) I think that semantics it is something like the native metavocabulary of reason relations—and so, of *roles* w/res to reason relations.
- d) Ulf describes Dan's semantics as an *abstract* specification of rational form.
What it abstracts *from* is the matter: subjective or objective, the activities by which the practitioners who deploy or use a vocabulary *take* claimables to be true and the worldly states that *make* those claimables true.

(Much) more on this issue in later weeks.

IV. Introduction to Implication Space Semantics

In “Semantic Inferentialism and Logical Expressivism” (Chapter 1 of *Articulating Reasons*) I introduce an inferentialist alternative to representationalist orders of semantic explanation.

Already in the opening session of this seminar I floated the idea that **the most basic characterization of claimables**—the **contents** of or meanings expressed by declarative sentences, paradigmatically in their assertional uses—**is as roles in reason relations of implication and incompatibility**.

For it is those reason relations that determine what is a reason for or against those claimables, and what they are reasons for or against—and thereby, their practical significance in rationally **defending** and **challenging** doxastic commitments.

The idea of understanding the *content* expressed by declarative sentences in terms of their role in implications and incompatibilities is the idea behind semantic inferentialism.

We saw how *logical* vocabularies can usefully be understood as a distinctive kind of metavocabulary with the expressive power to make reason relations explicit in extensions of arbitrary base vocabularies.

That is the idea of logical expressivism.

We saw how representationalist model-theoretic semantic metavocabularies, from primitive bivalent truth-functional semantics for classical logic to the sophisticated hyperintensional truthmaker semantics of the present day, can be understood as having as *a* if not *the* principal task, determining implication and incompatibility relations among the sentences they interpret.

All this is, I think, genuinely illuminating about the relations among the three vertices of the triangle of rational metavocabularies whose center is reason relations, in the mandala that is the orienting symbol of this enterprise.

From the beginning, however, **the most sought-after prize of the inferentialist program, its grail, “that one, far-off, divine event toward which the whole creation moves” has been a *direct specification* of the content of the claimables expressed by declarative sentences in terms of the relations of implication and incompatibility that they stand in to one another.**

To fulfill this dream, the inferentialist account of content in terms of reason relations must be as flexible, expressively powerful, and mathematically tractable as the best representational model-theoretic semantic specifications of content.

To descend to low autobiographical considerations, I can report that I have thought intensively about this topic and worked hard on this project for my whole philosophical career, beginning already in my dissertation. I feel like I learned a lot along the way—but a fully satisfactory inferentialist formal semantics eluded me.

That is exactly what Dan has achieved with his formal implication-space semantics.

It is what we inferentialists have always dreamed about.

It is a true inferentialist *conceptual role semantics* (CRS)—generically, as Harman, Field, and Boghossian use that term.

These are four three important large-scale features of Dan Kaplan’s implication-space semantics:

1. The *universe* from which the semantic interpretants of sentences is drawn—what in the four semantic frameworks we looked at last time plays the role played by the set of two truth-values, the set of all relational structures that are potential models of sentences of a vocabulary with a specified lexicon, the set of all possible worlds with an accessibility relation, and the mereologically structured set of all states (possible and impossible)—is a set of *candidate implications*.

We inferentialists had always thought that the *semantic interpretants* of sentences should be something like sets of implications—even that, as Dan has it, they should be *pairs* of sets of implications, corresponding to the inferential role the interpreted sentence plays as a *premise* and the inferential role the interpreted sentence plays as a *conclusion* in implications.

It was a huge conceptual step forward to *start* the process, not by interpreting *sentences*, but to have not only the semantic *interpretants*, but also what is semantically *interpreted*, consist to begin with of *implications* rather than *sentences*. (“It’s implications all the way down.”)

Candidate implications are ordered pairs of sets of sentences Γ, Δ (drawn from the lexicon of a base vocabulary). We can ask of each such $\langle \Gamma, \Delta \rangle$ whether $\Gamma \sim \Delta$.

The only structure on that universe is a distinguished set **I** of *good* implications, which answers that question.

The intended interpretation of the apparatus is that $\Gamma \sim \Delta$ just in case $\langle \Gamma, \Delta \rangle \in \mathbf{I}$.

There is also a mereological structure on this universe of candidate implications, though we will call the operation “adjunction” rather than “fusion.”

But where in specifying the universe of his semantics Fine *stipulates* the mereological structure as an *additional* element of structure, over and above the set of states—as the *accessibility* relation is an *additional* element of structure, over and above the set of possible worlds—Kaplan appeals to the set-theoretic structure of the points of his universe (the implication space) to *define* the adjunction monoid that, as we will see, does the work in his implication-space semantics that fusion does in Fine’s truthmaker semantics.

2. The *modal* character of the *v-function* that Dan defines on the implication space universe before we get to assigning *semantic interpretants* of sentences codifies an intuitively appealing and philosophically suggestive feature of implications, including candidates that are not good implications as they stand. For implications that are already good, the *v-function* assigns them, as their value, their *range of subjunctive robustness*. It assigns them all the *additional premises*

that, when playing the role of collateral commitments or auxiliary hypotheses *would not infirm* the implication—would not defeat it, in the sense of turning it from a good implication into a bad one. (A corresponding role is played by additions on the side of conclusions or consequences.) For candidate implications that are *not* good as they stand, where the conclusion does *not* follow from the premises (where commitment to accept all the premises and reject all the conclusions would *not* put one “out of bounds”), the ν -function specifies what one would need to add to that candidate implication to *make* it a good one.

Before looking at the details (I won’t say “intuitively,” because we are so deep in the theoretical woods at this point), this is just what we would want to know about a candidate implication beyond knowing whether it itself was good or bad: what good implications are in its neighborhood or vicinity, and how it is related to those good ones that are “nearby.”

In his classic essay “If, So, and Because,” Ryle says that any implication comes with a certain claim to generality. “Today is Wednesday, so tomorrow will be Thursday,” for instance, is not just about *these two days* (October 26 and 27 of 2022).

What Abelard calls “consequences *hic et nunc*,” implications that are so fragile that changing *any* feature of them, for instance, adding *any* further specification, infirms them, makes them no longer good, are so fragile as to be barely intelligible and of no practical use.

A large part of what we seek to understand about a conditional codifying an implication is precisely its *range of subjunctive robustness*: what one can change, what additional circumstances can be included, without undercutting or infirming the implication.

The hungry lioness would chase the wounded antelope if it were Thursday rather than Wednesday, if the beetle on a branch of a tree two miles away were slightly closer to the trunk of the tree, and if the temperature were 1/10th of a degree colder, but not if she were struck by lightning, the antelope suddenly flew off, the temperature dropped 200 degrees, and so on. This is the information that the ν -function encodes, by assigning to every candidate implication it applies to as an argument the set of all the additions that would keep it good if it is good, or make it good if it is not.

3. Bipolar, bilateral character of semantic interpretants of sentences. Here implication-space semantics can do what Fine’s truthmaker semantics can do using the two elements of the ordered pairs. Corresponding to truthmakers and falsemakers (verifiers and falsifiers) in Fine’s setting, we have premissory and conclusory roles. Restall-Ripley bilateralism shows us that and how these correspond: the premissory side of a sequent is the truth-maker side—the side of *assertion* (=taking-true) or commitments to *accept*—and the conclusory side is the falsemaker side—the side of *denial* (=taking-false) or commitments to *reject*.

These three points correspond to the phases of Dan’s semantic interpretation:

- 1) Universe of candidate implications, partitioned into good and bad ones (cf. Fine: universe of states partitioned into possible and impossible ones).

- 2) Define adjunction operation (rather than *stipulate* it, as Fine must do with fusion).
- 3) Define v -functions.
- 4) Define bilateral semantic interpretants of sentences, which again, do not need to be *stipulated*, as they do with Fine.
- 5) Use those semantic interpretants to specify the functional roles of sentences play in reason relations. First application and demonstration of usefulness of the semantics is completeness proof for NM-MS with arbitrary open-structured base vocabularies.

A key point is that because Dan approaches *semantics* from the perspective of an expressivist view of *logic*, where the point is to *extend* a base vocabulary, elaborating the new reason relations from the old ones, he starts his semantics from a base vocabulary, whose reason relations now take the form of the implication space with its partition into good and bad implications. This lets him *define* everything that Fine has to *stipulate* as additional structure:

- i) the commutative monoid that confers mereological structure, and
- ii) the semantic interpretation function that assigns sentences bipolar *pairs* of sets of elements of the underlying universe.

4. Then, Dan defines and generalizes the notions of premissory role and conclusory role of sentences. These articulate what is often called the “*external logic*” of reason relations, by which is meant a distinctive class of *metainferences* that the “*internal logic*”—the logic across the turnstile—induces.

This account illuminates the relations between our account and the projection of the phenomena Dan describes into the three-valued logics K3 and LP.

K3 shows up as the logic of truthmakers: more deeply, of *premissory metainferences*.

LP shows up as the logic of falsmakers: more deeply, of *conclusory metainferences*.